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The complete structure of the WG_2 algebra and its BRST quantization

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Abstract

The complete structure of the WG_2 algebra is obtained from an explicit realization by an abstract Virasoro algebra and a free boson field. We then construct its BRST operator and find a seven-parameter family of nilpotent BRST operators. These free parameters are related to the canonical transformations of the ghost, antighost fields which leave the total stress-energy tensor and the antighost field b invariant.

1 Introduction

Theres exist only three generic nonlinear extension of the Virasoro algebra by a single spin-s ($s > 2$) field. These algebras are related to the rank 2 Lie algebras. Two of them, W_3 and WB_2, related to the A_2 and B_2 ($= C_2$) algebras, are well-studied in the literature [1,2]. The other one, WG_2, related to the G_2 algebra, is a very complicated algebra. The structure of WG_2 is known by solving the bootstrap constraints or the Jacobi identities [3–5]. No explicit relization is known for WG_2. In application, it is important to know some realizations of the algebra. On the other hand, the authors of [6] starts directly from two free boson fields to formulate string theoy with higher spin extension of the Virasoro algebra. As emphasied in [7,8], this may give rise to results which are not true for abstract W-algebras. So it is important to study things using only the abstract W-algebras. In this paper we will first derive the complete structure of the WG_2 algebra. We use only an abstract Virasoro algebra and a free boson field to construct an explicit realization of WG_2. Because the original Virasoro algebra and the boson field satisfy the Jacobi identities, the consistency of the WG_2 algebra is automatically guaranteed. Having the abstract WG_2 algebra in hand, we then forget its realization and study its BRST quantization. To simplify the construction of

the BRST operator, we require that $\{Q, b(w)\}$ gives the total stress-energy tensor. Even with this restriction, the BRST operator is not unique and has seven free parameters. We will show that these parameters are related to the canonical transformation of the ghost, antighost fields which leave the total stress-energy tensor and the antighost field $b(z)$ invariant.

2 The WG_2 algebra

Let's start from the basic fields. ϕ is a free boson field. T_1 is the stress-energy tensor for an abstract Virasoro algebra. The basic OPEs are

$$\partial\phi(z)\partial\phi(w) \sim -\frac{1}{(z-w)^2}, \tag{1}$$

$$T_1(z)T_1(w) \sim \frac{\tilde{c}/2}{(z-w)^4} + \left(\frac{2}{(z-w)^2} + \frac{1}{z-w}\partial_w\right) T_1(w). \tag{2}$$

From these fields we can construct a new stress-energy tensor and a spin-6 field as follows:

$$T = T_1 - \frac{1}{2}(\partial\phi)^2 + a_0\partial^2\phi, \tag{3}$$

$$W = x_1(\partial\phi)^4 T_1 + x_2(\partial\phi)^6 + (27 \text{ more terms}). \tag{4}$$

By using the basic OPEs of T_1 and $\partial\phi$, one then computes the OPEs for T and W. The requirement that T and W form a (nonlinear) WG_2 algebra fixes all the coefficient x's. The OPEs for the WG_2 algebra can then be derived. The final result is [1]

$$\begin{aligned} T(z)T(w) \sim{}& \frac{c/2}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \Lambda_1(w) + (z-w)^2\Lambda_2(w) \\ &+ (z-w)^4\Lambda_4(w) + (z-w)^6\Lambda_7(w), \end{aligned} \tag{5}$$

$$T(z)W(w) \sim \frac{6}{(z-w)^2}W(w) + X_1(w) + (z-w)X_4(w) + (z-w)^2X_2(w), \tag{6}$$

$$W(z)W(w) \sim \frac{c/6}{(z-ww)^{12}} + \frac{2T(w)}{(z-w)^{10}} + \frac{b_1\Lambda_1(w)}{(z-w)^8}$$

[1] setting all derivatives of primary and quasi primary fields to 0, see the notation in [9].

$$
\begin{aligned}
&+\frac{1}{(z-w)^6}(b_2\Lambda_2(w)+b_3\Lambda_3(w)+c_0W(w))\\
&+\frac{1}{(z-w)^4}(b_4\Lambda_4(w)+b_5\Lambda_5(w)+b_6\Lambda_6(w)+c_1X_1(w))\\
&+\frac{1}{(z-w)^2}\left(\sum_{i=7}^{10}b_i\Lambda_i(w)+c_2X_2(w)+c_3X_3(w)\right),
\end{aligned}
\tag{7}
$$

where some quasi primary fields are defined in the above in terms of the nonsingular terms in OPEs and the rest are defined as follows:

$$
\begin{aligned}
T(z)\Lambda_1(w) &\sim (\text{singular terms})+\Lambda_3(w)+(z-w)^2\Lambda_5(w)\\
&\quad+(z-w)^3\Lambda_{11}(w)+(z-w)^4\Lambda_8(w), && (8)\\
T(z)\Lambda_3(w) &\sim (\text{singular terms})+\Lambda_6(w)+(z-w)\Lambda_{12}(w)+(z-w)^2\Lambda_9(w), && (9)\\
T(z)\Lambda_8(w) &\sim (\text{singular terms})+\Lambda_{10}(w), && (10)\\
T(z)X_1(w) &\sim (\text{singular terms})+X_3(w). && (11)
\end{aligned}
$$

Here Λ_{11} and $\Lambda_{12}=-8\,\Lambda_{11}/5$ are spin-9 quasi primary fields which don't appear in the OPEs of the WG_2 algebra. The various coefficients appearing in (7) are given as follows:

$$
\begin{aligned}
b_1 &= \frac{62}{22+5\,c}, \qquad b_2 = \frac{-740+17\,c}{(-1+2\,c)\,(68+7\,c)},\\
b_3 &= \frac{80\,(35+139\,c)}{3\,(-1+2\,c)\,(22+5\,c)\,(68+7\,c)},\\
b_4 &= \frac{-6450504-5184658\,c+17697\,c^2+1630\,c^3}{25\,(-1+2\,c)\,(46+3\,c)\,(3+5\,c)\,(68+7\,c)},\\
b_5 &= \frac{6310112-46423496\,c-4678806\,c^2+121395\,c^3}{15\,(-1+2\,c)\,(46+3\,c)\,(3+5\,c)\,(22+5\,c)\,(68+7\,c)},\\
b_6 &= \frac{8\,(-2179+57652\,c+22992\,c^2)}{(-1+2\,c)\,(46+3\,c)\,(3+5\,c)\,(22+5\,c)\,(68+7\,c)},\\
b_7 &= \frac{-3422146368-1675894344\,c-7636990\,c^2+268425\,c^3+6625\,c^4}{25\,(-1+2\,c)\,(46+3\,c)\,(3+5\,c)\,(68+7\,c)\,(232+11\,c)},\\
b_8 &= \frac{2\,(25916384896-45359110192\,c-7010455500\,c^2+6482100\,c^3+2541375\,c^4)}{225\,(-1+2\,c)\,(46+3\,c)\,(3+5\,c)\,(22+5\,c)\,(68+7\,c)\,(232+11\,c)},\\
b_9 &= \frac{8\,(-1089457712-16862376526\,c-1015716375\,c^2+28458300\,c^3)}{585\,(-1+2\,c)\,(46+3\,c)\,(3+5\,c)\,(22+5\,c)\,(68+7\,c)\,(232+11\,c)},\\
b_{10} &= \frac{32\,(15707+874936\,c+172800\,c^2)}{(-1+2\,c)\,(46+3\,c)\,(3+5\,c)\,(22+5\,c)\,(68+7\,c)\,(232+11\,c)},\\
c_0^2 &= \frac{400(47+2\,c)^2(516+13\,c)^2(2+c)(c^2-388c+4)}{3(2c-1)(3c+46)(3c+286)(5c+3)(5c+22)(7c+68)(11c+232)},
\end{aligned}
$$

$$
\begin{aligned}
c_1 &= \frac{186c_0}{13c+516c}, \qquad c_2 = \frac{9(13c-694)c_0}{130(c+2)(c+47)}, \\
c_3 &= \frac{12(572c+2089)c_0}{5(c+2)(c+47)(516+13c)}.
\end{aligned} \tag{12}
$$

We have checked that the above structure of the WG_2 algebra is in agreement with that given in [3].

3 The BRST quantization of the WG_2 algebra

Now we study the BRST quantization of the WG_2 algebra. Following the standard procedure we introduce ghost, antighost paris $(c(z), b(z))$ and $(\delta(z), \alpha(z))$ for $T(z)$ and $W(z)$ respectively. These ghost anti-ghost fields have spins $(-1, 2)$ and $(-5, 6)$ and their mode expansions are as follows

$$
c(z) = \sum_n c_n z^{-n+1}, \qquad b(z) = \sum_n b_n z^{-n-2}, \tag{13}
$$

$$
\delta(z) = \sum_n \delta_n z^{-n+5}, \qquad \alpha(z) = \sum_n \alpha_n z^{-n-6}. \tag{14}
$$

These modes satisfy the usual anti-commutation relations which can be derived from the following OPEs

$$
c(z)b(w) \sim \frac{1}{z-w}, \tag{15}
$$

$$
\delta(z)\alpha(w) \sim \frac{1}{z-w}. \tag{16}
$$

The other OPEs are all 0 (nonsingular).

Because of the complexity with normal ordering we will not use mode expansions. All our calculation are done with (the holomorphic) fields. The normal ordering for the ghost anti-ghost fields are such that the following equations are true

$$
c(z)b(w) = \frac{1}{z-w} + :c(z)b(w):, \tag{17}
$$

$$
\delta(z)\alpha(w) = \frac{1}{z-w} + :\delta(z)\alpha(w):. \tag{18}
$$

This is possible because all these fields are free fields.

With all the above knowledge, we now construct the quantum BRST operator. One way to start is to construct the corresponding classical BRST operator

[10]. The quantum BRST operator is then assumed to be the same form as the classical one with possible renormalization of some coefficients and addition of some zero mode terms due to normal ordering. By imposing the nilpotent condition, one would determine all these coefficients. For linear algebras this route is quite successful. The same strategy has been applied to W_3 [10] and in [11] to a class of quadratic non-linear algebra. But the simplicity of this construction doesn't apply to more complicated nonlinear algebras, such as WB_2 and W_4 [8,12].

We will follow the same strategy used in [8] for the BRST quantization of the WB_2 and W_4 algebras. The BRST operator is the contour integration of a spin-1 current $j(z)$ with ghost number 1. To simplify our calculations, we require that the (anti-) commutator of the BRST operator Q with the stress-energy tensor antighost $b(z)$ gives the total stree-energy tensor:

$$\begin{aligned}\{Q, b(z)\} = T_{\rm tot} \equiv T(z) + 2c'(z)b(z) + c(z)b'(z) \\ + 6\delta'(z)\alpha(z) + 5\delta(z)\alpha'(z)\,.\end{aligned} \tag{19}$$

This fixes the dependence of the BRST current $j(z)$ on the ghost field $c(z)$ to the the following form

$$j(z) =: c(z)(T(z) + c'(z)b(z) + 6\delta'(z)\alpha(z) + 5\delta(z)\alpha'(z)) : + \cdots. \tag{20}$$

We will group the rest terms by their (δ,α)-ghost number. By ghost number and spin counting, there are possible terms with (δ,α)-ghost number from 1 to 5. The ansatz for the $((\delta,\alpha)$-) ghost number 1 terms is

$$\begin{aligned}j_1 = \delta\,(aW + m_1 T^2\delta'\alpha + m_2 T''\delta'\alpha + m_3 T'\delta'\alpha' + m_4 T\delta'\alpha'' \\ + m_5 T\delta^{(3)}\alpha + m_6\delta'\alpha^{(5)} + m_7\delta^{(3)}\alpha^{(3)} + m_8\delta^{(5)}\alpha' + m_9\delta''\delta'\alpha'\alpha).\end{aligned} \tag{21}$$

Here a is an arbitrary constant which set the normalization for the δ ghost. It is set to be $a = \sqrt{2560504830}$. j_2, j_3, j_4 and j_5 have 80, 124, 51 and 4 terms respectively. We will not give their explicit form here. After we have solved the nilpotent condition, we will give a simplified form of the BRST operator in an Appendix. Writing the BRST operator as the sum of various (δ,α) ghost number terms:

$$Q = Q_0 + Q_1 + Q_2 + Q_3 + Q_4 + Q_5, \tag{22}$$

$$Q_i = \oint_0 [\,\mathrm{dz}]\mathrm{j_i(z)}, \tag{23}$$

the nilpotent condtion $Q^2 = 0$ becomes

$$0 = Q_0^2, \tag{24}$$
$$0 = \{Q_0, Q_1\}, \tag{25}$$
$$0 = \{Q_0, Q_2\} + Q_1^2, \tag{26}$$
$$0 = \{Q_0, Q_3\} + \{Q_1, Q_2\}, \tag{27}$$
$$0 = \{Q_0, Q_4\} + \{Q_1, Q_3\} + Q_2^2, \tag{28}$$
$$0 = \{Q_0, Q_5\} + \{Q_1, Q_4\} + \{Q_2, Q_3\}, \tag{29}$$
$$0 = \{Q_1, Q_5\} + \{Q_2, Q_4\} + Q_3^2, \tag{30}$$
$$0 = \{Q_2, Q_5\} + \{Q_3, Q_4\}, \tag{31}$$
$$0 = \{Q_3, Q_5\} + Q_4^2, \tag{32}$$
$$0 = \{Q_4, Q_5\}, \tag{33}$$
$$0 = Q_0^5, \tag{34}$$
$$\tag{35}$$

The first equation gives the critical central charge $c = 388$. The second equation (25) is satisfied only for $m_i = 0, i = 1, \cdots, 9$. We then solve equations (26) and (27) together to found a 7-parameter solution for the 204 coefficients in Q_2 and Q_3. The real time consuming part of the calculations is to check eqs. (28)–(30). The rest eqs. (31)–(34) are satisfied automatically by ghost, antighost counting. Because the calculations will take too long a time by simply using the `OPEdefs.m` Mathematica package [13], one must write some other programmes to do the calculation. Let us say a few words about how we actually did the calculations.

First it is a good idea to split the bosonic and fermionic part of every terms. Because we knew that only lower spin (≤ 11) bosonic fields could appear in Q^2, we can expand all the OPEs (including also nonsingular terms) to a certain degree approparitely. The OPEs for quasi primary fields are also needed. These can be obtained easily by using the OPEs and the definitions for quasi primary fields given in (5) to (11). The OPEs with the fermionic part are not so easy. One can just try a simple example by computing the OPEs of $B_i(z) =: c^{(i)}(z) \cdots c'(z)\, c(z)\, b^{(i)}(z) \cdots b'(z)\, b(z) :$ with itself. The time needed grows quite rapidly with i. Because all the fermionic ghost fields are free fields, we can use eqs. (17) and (18) to do all the possible contractions and then obtain the OPEs by expanding all fields around w. Actually, most of the computing time are used in this Laurent expansion. I have written a simple programme to do all these things but only for these free fermionic ghosts. For the problem in hand, the computations take much short time and it is possible to finish all the calculation in about two weeks. Here is the result: all the eqs. (28)–(30) are satisfied and the rest 55 coefficients in Q_4 and Q_5 are also found. The complete solution is quite long because there are seven free parameters. After explaining the meaning of the 7 free parameters in Q and also puting some terms to 0, I will give an explicit solution in the Appendix.

4 The canonical transformations of the ghost, antighost fields

The canonical transformations of the ghost, antighost fields discussed in [12,8] are similar transformations. One easily verifies that the 3-parameter and 7-parameter canonical transformations for WB_2 and W_4 ghost, antighost fields are the following transformations:

$$(\text{field}) \longrightarrow \mathrm{e}^{S}\,(\text{field})\,e^{-S} = \sum_{n=0}^{\infty} \frac{1}{n!}\,\underbrace{[S, \cdots, [S}_{n \text{ times}}, (\text{field})] \cdots], \tag{36}$$

where S is the the contour integration of a spin-1 current of ghost number 0:

$$S = S_{WB_2} = \oint [\,\mathrm{dz}]\,(\mathrm{c}_1 \mathrm{b}'\mathrm{b}\delta''\delta + \mathrm{c}_2 \mathrm{c}\,\mathrm{b}'\,\mathrm{b}\,\delta + \mathrm{c}_3\,\mathrm{b}\,\delta'\,\delta\,\alpha\,)\,, \tag{37}$$

for WB_2 and

$$S = S_{WB_2} + \oint [\,\mathrm{dz}]\,(\mathrm{c}_4\,\mathrm{b}\,\gamma'\,\beta\,\delta + \mathrm{c}_5\,\mathrm{b}\,\gamma\,\beta'\,\delta + \mathrm{c}_6\,\mathrm{b}\,\gamma\,\beta\,\delta' + \mathrm{c}_7\,\mathrm{b}'\,\mathrm{b}\,\gamma\,\beta\,\delta'\,\delta)\,, \tag{38}$$

for W_4 ghost, antighost fields. From these results, we learnt that in order to obtain all the possible canonical transformations, one simply writes down all the possible ghost number 0 spin-1 currents module total derivatives. Of course, we omit some of the simplest canonical transformations generated by the currents $c\,b$, $\delta\,\alpha$ and $b\,\delta''$. These are just simple rescaling and shifting of the ghost fields.

For WG_2 ghost, antighost field system, there are altogether 56 independent ghost number 0 spin-1 currents (including simple rescaling and shifting). One thing to be noticed here is that these currents can also involve the Virasoro field because the spin of the δ ghost is so low that there exists many ghost, antighost currents with very low spin. The canonical transformation for WG_2 is a 56-parameter family transformations. It is not difficult to obtain the full transformation group. The more interesting thing is the subgroup of canonical transformations which leave the total stress-energy tensor and the antighost $b(z)$ invariant. This subgroup is a 10-parameter group. Nevertheless 3 parameters are fixed by our ansatz for the BRST operator:

$$j(z) = c(z)T(z) + a\,\delta(z)W(z) + (3 \text{ or more ghost terms})\,. \tag{39}$$

Simple rescaling of $c(z)$ and $\delta(z)$ are not allowed. Also simple shifting of $c(z)$ by $\delta^{(4)}(z)$ is not allowed. This then leaves a 7-parameter subgroup of canonical transformations which leaves the total stress-energy tensor and the antighost

$b(z)$ invariant. This is why we obtain a seven-parameter family of nilpotent BRST operator for WG_2 algebra.

Now we give the explicit form of the seven parameter generator for the canonical transformations. We split it into 2 groups according the number of ghost antighost fields. We have

$$
\begin{aligned}
J_1 = & \, c_1\, T\, b''\, b'\, b\, \delta^{(3)}\, \delta''\, \delta + c_2\, T\, b''\, b'\, b\, \delta^{(4)}\, \delta'\, \delta + c_3\, T\, b^{(3)}\, b'\, b\, \delta^{(3)}\, \delta'\, \delta \\
& + c_4\, b''\, b'\, b\, \delta^{(3)}\, \delta''\, \delta'\, \delta\, \alpha - \left(6\, c_1 + \frac{28\, c_2}{3} - 6\, c_3\right) T^2\, b''\, b'\, b\, \delta''\, \delta'\, \delta \\
& - \left(\frac{2683\, c_1}{89} + \frac{4508\, c_2}{89} - \frac{2877\, c_3}{89}\right) T\, b^{(3)}\, b''\, b\, \delta''\, \delta'\, \delta \\
& + \left(\frac{901\, c_1}{89} + \frac{1521\, c_2}{89} - \frac{894\, c_3}{89}\right) T\, b^{(4)}\, b'\, b\, \delta''\, \delta'\, \delta \\
& + \left(\frac{421687906\, c_1}{666165} + \frac{745270691\, c_2}{666165} - \frac{159173993\, c_3}{222055} - \frac{193925\, c_4}{13972}\right) b''\, b'\, b\, \delta^{(4)}\, \delta''\, \delta' \\
& + \left(\frac{919625281\, c_1}{2664660} + \frac{407256149\, c_2}{666165} - \frac{1043606219\, c_3}{2664660} - \frac{207681\, c_4}{27944}\right) b''\, b'\, b\, \delta^{(4)}\, \delta^{(3)}\, \delta \\
& + \left(\frac{48729017\, c_1}{222055} + \frac{259636796\, c_2}{666165} - \frac{167000729\, c_3}{666165} - \frac{219601\, c_4}{41916}\right) b''\, b'\, b\, \delta^{(5)}\, \delta''\, \delta \\
& - \left(\frac{10612867\, c_1}{666165} + \frac{5834259\, c_2}{222055} - \frac{3663884\, c_3}{222055} + \frac{5603\, c_4}{69860}\right) b''\, b'\, b\, \delta^{(6)}\, \delta'\, \delta \\
& - \left(\frac{421687906\, c_1}{666165} + \frac{745270691\, c_2}{666165} - \frac{159173993\, c_3}{222055} - \frac{47109\, c_4}{3493}\right) b^{(4)}\, b'\, b\, \delta^{(3)}\, \delta''\, \delta \\
& - \left(\frac{27535487\, c_1}{222055} + \frac{98770929\, c_2}{444110} - \frac{63720561\, c_3}{444110} - \frac{3447\, c_4}{998}\right) b^{(4)}\, b'\, b\, \delta^{(4)}\, \delta'\, \delta \\
& + \left(\frac{2318203\, c_1}{7485} + \frac{1370416\, c_2}{2495} - \frac{879499\, c_3}{2495} - \frac{49407\, c_4}{6986}\right) b^{(4)}\, b^{(3)}\, b\, \delta''\, \delta'\, \delta \\
& - \left(\frac{2318203\, c_1}{9980} + \frac{1027812\, c_2}{2495} - \frac{2638497\, c_3}{9980} - \frac{148221\, c_4}{27944}\right) b^{(5)}\, b'\, b\, \delta^{(3)}\, \delta'\, \delta,
\end{aligned}
\tag{40}
$$

and

$$
\begin{aligned}
J_2 = & \, c_5\, T\, b'\, b\, \delta^{(3)}\, \delta' + c_6\, T\, b'\, b\, \delta^{(4)}\, \delta + c_7\, T\, b''\, b\, \delta''\, \delta' \\
& + \left(\frac{356406389669\, c_5}{29792560608} - \frac{1482203755439\, c_6}{148962803040} + \frac{1385455753\, c_7}{9930853536}\right) b'\, b\, \delta^{(4)}\, \delta'' \\
& + \left(\frac{111038273687\, c_5}{37240700760} + \frac{74234639611\, c_6}{186203503800} - \frac{981306029\, c_7}{12413566920}\right) b'\, b\, \delta^{(5)}\, \delta' \\
& - \left(\frac{1261098246353\, c_5}{446888409120} - \frac{3548929059731\, c_6}{2234442045600} - \frac{19561388891\, c_7}{148962803040}\right) b'\, b\, \delta^{(6)}\, \delta \\
& - \left(\frac{1406793619\, c_5}{164903472} - \frac{1324530053\, c_6}{164903472} - \frac{26925745\, c_7}{54967824}\right) b^{(3)}\, b\, \delta^{(3)}\, \delta' \\
& + \left(\frac{1004852585\, c_5}{329806944} - \frac{946092895\, c_6}{329806944} - \frac{19232675\, c_7}{109935648}\right) b^{(3)}\, b\, \delta^{(4)}\, \delta
\end{aligned}
$$

$$
\begin{aligned}
&-\Big(\frac{454062238\,c_5}{931017519}+\frac{2526875414\,c_6}{4655087595}+\frac{479486714\,c_7}{310339173}\Big)\,b'\,b\,\delta^{(3)}\,\delta''\,\delta\,\alpha\\
&-\Big(\frac{2541405917\,c_5}{931017519}+\frac{27844868791\,c_6}{4655087595}+\frac{284378101\,c_7}{310339173}\Big)\,b'\,b\,\delta^{(4)}\,\delta'\,\delta\,\alpha\\
&-\Big(\frac{7933248596\,c_5}{931017519}+\frac{16502801672\,c_6}{931017519}+\frac{1659334768\,c_7}{310339173}\Big)\,b''\,b\,\delta^{(3)}\,\delta'\,\delta\,\alpha\\
&-\Big(\frac{15221893295\,c_5}{620678346}+\frac{3102509207\,c_6}{620678346}+\frac{1025874619\,c_7}{206892782}\Big)\,b''\,b'\,\delta''\,\delta'\,\delta\,\alpha\\
&-\Big(\frac{5503700363\,c_5}{931017519}+\frac{20969565827\,c_6}{931017519}+\frac{1870488151\,c_7}{310339173}\Big)\,b^{(3)}\,b\,\delta''\,\delta'\,\delta\,\alpha\\
&+\Big(\frac{4474\,c_5}{4289}+\frac{60233\,c_6}{21445}-\frac{100\,c_7}{4289}\Big)\,T\,b''\,b\,\delta^{(3)}\,\delta\\
&+\Big(\frac{523297\,c_5}{137248}+\frac{417533\,c_6}{686240}+\frac{34175\,c_7}{137248}\Big)\,T\,b''\,b'\,\delta''\,\delta\\
&+\Big(\frac{125717\,c_5}{205872}+\frac{3729697\,c_6}{1029360}+\frac{29705\,c_7}{68624}\Big)\,T\,b^{(3)}\,b\,\delta''\,\delta\\
&+\Big(\frac{529217\,c_5}{137248}+\frac{423517\,c_6}{686240}+\frac{30975\,c_7}{137248}\Big)\,T\,b^{(3)}\,b'\,\delta'\,\delta\\
&-\Big(\frac{8959\,c_5}{68624}-\frac{770893\,c_6}{343120}-\frac{15855\,c_7}{68624}\Big)\,T\,b^{(4)}\,b\,\delta'\,\delta\\
&+\Big(\frac{1850\,c_5}{12867}+\frac{374\,c_6}{12867}-\frac{1000\,c_7}{12867}\Big)\,T^2\,b'\,b\,\delta''\,\delta\\
&+\Big(\frac{1110\,c_5}{4289}+\frac{1122\,c_6}{21445}-\frac{600\,c_7}{4289}\Big)\,T^2\,b''\,b\,\delta'\,\delta\\
&-\Big(\frac{9526\,c_5}{4289}+\frac{34\,c_6}{4289}-\frac{3990\,c_7}{4289}\Big)\,T\,b'\,b\,\delta''\,\delta'\,\delta\,\alpha. \qquad (41)
\end{aligned}
$$

By using these expressions, I have checked explictly that the 4 transformations associated with J_1 which change the δ ghost filed give exactly the form fo the BRST operator obtained by explicit computations. In particular one can use these canonical transformations to put all W-dependent terms in Q_3 and Q_4 to 0. After doing that, the BRST operator can be written as the sum of four anticommuting nilpotnet operators:

$$Q = \tilde{Q}_0 + c_1\tilde{Q}_1 + c_2\tilde{Q}_2 + c_3\tilde{Q}_3, \qquad \{\tilde{Q}_i, \tilde{Q}_j\} = 0. \qquad (42)$$

The dependence of the BRST operator on the rest three free parameters is linear. Also the rest canonical transformations associated with J_2 are just linear transformations. The explicit expression given in the Appendix is $\tilde{Q}_0$, arranged by their (δ, α) ghost number. The other three $\tilde{Q}_i$'s are $\tilde{Q}_0$ exact.

Appendix: The explicit expression of the BRST current

The BRST current of $\tilde{Q}_0$ contains various (δ, α)-ghost number terms. The 80 ghost number 2 terms are

$$
\begin{array}{llllll}
\frac{59673456869}{498960000} & b\delta^{(5)}\delta^{(4)} & \frac{31195928179}{207900000} & b\delta^{(6)}\delta^{(3)} & \frac{-3256883498089}{8731800000} & b\delta^{(7)}\delta'' \\
\frac{152213868677}{317520000} & b\delta^{(8)}\delta' & \frac{-152213868677}{571536000} & b\delta^{(9)}\delta & \frac{-5574877217}{81675000} & b\delta''\delta'\delta\alpha^{(5)} \\
\frac{-5574877217}{24502500} & b\delta^{(3)}\delta'\delta\alpha^{(4)} & \frac{-196813097}{1089000} & b\delta^{(3)}\delta''\delta\alpha^{(3)} & \frac{26502125533}{32670000} & b\delta^{(3)}\delta''\delta'\alpha'' \\
\frac{-16985555249}{45738000} & b\delta^{(4)}\delta'\delta\alpha^{(3)} & \frac{-932737741}{1663200} & b\delta^{(4)}\delta''\delta\alpha'' & \frac{1328840539}{1089000} & b\delta^{(4)}\delta''\delta'\alpha' \\
\frac{-52596629033}{54885600} & b\delta^{(4)}\delta^{(3)}\delta\alpha' & \frac{863380219}{871200} & b\delta^{(4)}\delta^{(3)}\delta'\alpha & \frac{-83896046221}{457380000} & b\delta^{(5)}\delta'\delta\alpha'' \\
\frac{-11539195369}{91476000} & b\delta^{(5)}\delta''\delta\alpha' & \frac{-553073339}{1815000} & b\delta^{(5)}\delta''\delta'\alpha & \frac{-195848181061}{274428000} & b\delta^{(5)}\delta^{(3)}\delta\alpha \\
\frac{-20406600331}{245025000} & b\delta^{(6)}\delta'\delta\alpha' & \frac{426766016729}{686070000} & b\delta^{(6)}\delta''\delta\alpha & \frac{-6972875060203}{16008300000} & b\delta^{(7)}\delta'\delta\alpha \\
\frac{19416854041}{16335000} & b\delta^{(3)}\delta''\delta'\delta\alpha'\alpha & \frac{57029639797}{55440000} & Tb\delta^{(4)}\delta^{(3)} & \frac{-12364869680501}{9147600000} & Tb\delta^{(5)}\delta'' \\
\frac{3220505762981}{9147600000} & Tb\delta^{(6)}\delta' & \frac{-45278977634813}{192099600000} & Tb\delta^{(7)}\delta & \frac{-69611369737}{52272000} & Tb'\delta^{(4)}\delta'' \\
\frac{80479622431}{1524600000} & Tb'\delta^{(5)}\delta' & \frac{-7902513471619}{9147600000} & Tb'\delta^{(6)}\delta & \frac{-710049471547}{152460000} & Tb''\delta^{(3)}\delta'' \\
\frac{4488333158933}{1829520000} & Tb''\delta^{(4)}\delta' & \frac{-4807942668217}{1829520000} & Tb''\delta^{(5)}\delta & \frac{48214065211}{34303500} & Tb^{(3)}\delta^{(3)}\delta' \\
\frac{-5774383084717}{1829520000} & Tb^{(3)}\delta^{(4)}\delta & \frac{459106098581}{228690000} & Tb^{(4)}\delta''\delta' & \frac{-3727403771143}{1372140000} & Tb^{(4)}\delta^{(3)}\delta \\
\frac{-2574566500831}{2286900000} & Tb^{(5)}\delta''\delta & \frac{-432477750757}{1143450000} & Tb^{(6)}\delta'\delta & \frac{97387696081}{65340000} & Tb\delta''\delta'\delta\alpha^{(3)} \\
\frac{14580671833}{4356000} & Tb\delta^{(3)}\delta'\delta\alpha'' & \frac{7730567377}{16940000} & Tb\delta^{(3)}\delta''\delta\alpha' & \frac{-524120096617}{152460000} & Tb\delta^{(3)}\delta''\delta'\alpha \\
\frac{84825821821}{22869000} & Tb\delta^{(4)}\delta'\delta\alpha' & \frac{606431695753}{457380000} & Tb\delta^{(4)}\delta''\delta\alpha & \frac{18452903729}{15246000} & Tb\delta^{(5)}\delta'\delta\alpha \\
\frac{85278024871}{16940000} & Tb'\delta''\delta'\delta\alpha'' & \frac{334254453767}{45738000} & Tb'\delta^{(3)}\delta'\delta\alpha' & \frac{-107759818127}{457380000} & Tb'\delta^{(3)}\delta''\delta\alpha \\
\frac{4751408701}{1089000} & Tb'\delta^{(4)}\delta'\delta\alpha & \frac{23294487541}{4620000} & Tb''\delta''\delta'\delta\alpha' & \frac{541682200891}{152460000} & Tb''\delta^{(3)}\delta'\delta\alpha \\
\frac{718566565093}{457380000} & Tb^{(3)}\delta''\delta'\delta\alpha & \frac{197\,a}{900} & Wb\delta''\delta' & \frac{-9\,a}{100} & Wb\delta^{(3)}\delta \\
\frac{-98\,a}{2475} & Wb'\delta''\delta & \frac{-9\,a}{100} & Wb''\delta'\delta & \frac{-4064194451083}{2744280000} & T^2b\delta^{(3)}\delta'' \\
\frac{2921472697793}{5488560000} & T^2b\delta^{(4)}\delta' & \frac{-172159457821}{1097712000} & T^2b\delta^{(5)}\delta & \frac{-46505121533}{1372140000} & T^2b'\delta^{(3)}\delta' \\
\frac{-30539070601}{152460000} & T^2b'\delta^{(4)}\delta & \frac{284730936659}{152460000} & T^2b''\delta''\delta' & \frac{-994188795821}{1372140000} & T^2b''\delta^{(3)}\delta \\
\frac{-214406574031}{2744280000} & T^2b^{(3)}\delta''\delta & \frac{-118177330583}{784080000} & T^2b^{(4)}\delta'\delta & \frac{183119385787}{228690000} & T^2b\delta''\delta'\delta\alpha' \\
\frac{19456666073}{32670000} & T^2b\delta^{(3)}\delta'\delta\alpha & \frac{5951595949}{5082000} & T^2b'\delta''\delta'\delta\alpha & \frac{-103\,a}{1650} & TWb\delta'\delta \\
\frac{407431932661}{457380000} & T''Tb\delta''\delta' & \frac{-29420636167}{114345000} & T''Tb\delta^{(3)}\delta & \frac{-12068938999}{114345000} & T''Tb'\delta''\delta \\
\frac{-231200488649}{457380000} & T''Tb''\delta'\delta & \frac{-77520792869}{914760000} & T^{(4)}Tb\delta'\delta & \frac{1904078111}{5488560} & T^3b\delta''\delta' \\
\frac{-9198368491}{68607000} & T^3b\delta^{(3)}\delta & \frac{-8926920587}{137214000} & T^3b'\delta''\delta & \frac{-28861261637}{171517500} & T^3b''\delta'\delta \\
\frac{-12589653817}{114345000} & T''T^2b\delta'\delta & \frac{-157515137}{6534000} & T^4b\delta'\delta. & &
\end{array}
$$

There are 124 possible ghost number 3 terms. Some of them are fixed by using the seven-parameter canonical transformations. The rest 115 terms are as follows:

$$
\begin{array}{llll}
\frac{-7934109042500138719113269}{153379167618600000} & b'b\delta^{(5)}\delta^{(4)}\delta'' & \frac{-1667368584022693256429372999}{2530756265706900000} & b'b\delta^{(6)}\delta^{(3)}\delta'' \\
\frac{-16666473279072934299304361}{209153410389000000} & b'b\delta^{(6)}\delta^{(4)}\delta' & \frac{-568989949468038155157773777}{379613439856035000000} & b'b\delta^{(6)}\delta^{(5)}\delta \\
\frac{-3996922374558882014726837201}{590509795331610000000} & b'b\delta^{(7)}\delta^{(3)}\delta' & \frac{-239950378585579439331129563}{84358542190230000000} & b'b\delta^{(7)}\delta^{(4)}\delta \\
\frac{-1397471914359077916986248427}{708611754397932000000} & b'b\delta^{(8)}\delta''\delta' & \frac{-177426073968190731181444427}{96628875599718000000} & b'b\delta^{(8)}\delta^{(3)}\delta \\
\frac{-1971041306949715186374210191}{3543058771989660000000} & b'b\delta^{(9)}\delta''\delta & \frac{-233732536262351277656915033}{3543058771989660000000} & b'b\delta^{(10)}\delta'\delta \\
\frac{34024243484459847510641269}{303690751884828000000} & b^{(3)}b\delta^{(4)}\delta^{(3)}\delta'' & \frac{162845556695764420711023713}{50615125314138000000} & b^{(3)}b\delta^{(5)}\delta^{(3)}\delta' \\
\frac{14319989479529649797115247}{151845375942414000000} & b^{(3)}b\delta^{(5)}\delta^{(4)}\delta & \frac{662170963749315819119972261}{379613439856035000000} & b^{(3)}b\delta^{(6)}\delta''\delta' \\
\frac{1152799767141684025639421809}{759226879712070000000} & b^{(3)}b\delta^{(6)}\delta^{(3)}\delta & \frac{616810504739088900353741563}{885764692997415000000} & b^{(3)}b\delta^{(7)}\delta''\delta \\
\frac{579710950851494769940193353}{5314588157984490000000} & b^{(3)}b\delta^{(8)}\delta'\delta & \frac{-84026779312214353332827993}{2530756265706900000000} & b^{(5)}b\delta^{(4)}\delta''\delta' \\
\frac{-2777065577483826084816919}{126537813285345000000} & b^{(5)}b\delta^{(4)}\delta^{(3)}\delta & \frac{-60139791185852666093989963}{2530756265706900000000} & b^{(5)}b\delta^{(5)}\delta''\delta \\
\frac{-3559836953868589034092829}{665988490975500000000} & b^{(5)}b\delta^{(6)}\delta'\delta & \frac{20916119887299280076956861}{664323519748061250000} & b^{(7)}b\delta^{(3)}\delta''\delta \\
\frac{3736177184815092491406959}{295254897665805000000} & b^{(7)}b\delta^{(4)}\delta'\delta & \frac{-1236099637605531047159317}{579773253598308000000} & b^{(9)}b\delta''\delta'\delta \\
\frac{305543680943510417689229}{25307562657069000000} & b'b\delta^{(4)}\delta^{(3)}\delta''\delta'\alpha & \frac{310663406860065680073211}{25307562657069000000} & b'b\delta^{(5)}\delta^{(3)}\delta''\delta\alpha \\
\frac{119522985133420812681049}{12653781328534500000} & b'b\delta^{(5)}\delta^{(4)}\delta'\delta\alpha & \frac{61343907267104214014431}{15817226660668125000} & b'b\delta^{(6)}\delta^{(3)}\delta'\delta\alpha \\
\frac{-2713550593929745519201499}{126537813285345000000} & b'b\delta^{(7)}\delta''\delta'\delta\alpha & \frac{618766950761853728954431}{12653781328534500000} & b''b\delta^{(4)}\delta^{(3)}\delta''\delta\alpha \\
\frac{1522973603977375002414923}{25307562657069000000} & b''b\delta^{(5)}\delta^{(3)}\delta'\delta\alpha & \frac{-168951016933549645912819}{3329942454877500000} & b''b\delta^{(6)}\delta''\delta'\delta\alpha \\
\frac{967690718032038057905549}{8435854219023000000} & b''b'\delta^{(4)}\delta^{(3)}\delta'\delta\alpha & \frac{5723656916482740608789}{200853671881500000} & b''b'\delta^{(5)}\delta''\delta'\delta\alpha \\
\frac{447242299447329584285243}{3615366093867000000} & b^{(3)}b\delta^{(4)}\delta^{(3)}\delta'\delta\alpha & \frac{-12258051644696011305541}{1150343757139500000} & b^{(3)}b\delta^{(5)}\delta''\delta'\delta\alpha \\
\frac{133492916436566519985997}{665988490975500000} & b^{(3)}b'\delta^{(4)}\delta''\delta'\delta\alpha & \frac{6023609758692629765467}{66951223960500000} & b^{(3)}b''\delta^{(3)}\delta''\delta'\delta\alpha \\
\frac{333967635166384940431373}{5061512531413800000} & b^{(4)}b\delta^{(4)}\delta''\delta'\delta\alpha & \frac{4259543507166022741129841}{25307562657069000000} & b^{(4)}b'\delta^{(3)}\delta''\delta'\delta\alpha \\
\frac{3714661861828725086984107}{126537813285345000000} & b^{(5)}b\delta^{(3)}\delta''\delta'\delta\alpha & \frac{13800716658064624117063}{4217927109511500000} & Tb'b\delta^{(4)}\delta^{(3)}\delta'' \\
\frac{213773474384983500107419}{50615125314138000000} & Tb'b\delta^{(5)}\delta^{(3)}\delta' & \frac{-1020068015588230739429}{3163445332133625000} & Tb'b\delta^{(5)}\delta^{(4)}\delta \\
\frac{130304558569038116783117}{23006875142790000000} & Tb'b\delta^{(6)}\delta''\delta' & \frac{-359689583127547857672293}{2530756265706900000000} & Tb'b\delta^{(6)}\delta^{(3)}\delta \\
\frac{179010496901458732144219}{80524062999765000000} & Tb'b\delta^{(7)}\delta''\delta & \frac{837309222531646562877127}{885764692997415000000} & Tb'b\delta^{(8)}\delta'\delta \\
\frac{-5630710848055468467773}{8435854219023000000} & Tb''b\delta^{(4)}\delta^{(3)}\delta' & \frac{-178356879627206203221467}{33743416876092000000} & Tb''b\delta^{(5)}\delta''\delta' \\
\frac{270487127804498569503773}{101230250628276000000} & Tb''b\delta^{(5)}\delta^{(3)}\delta & \frac{6022597969889820098814047}{506151253141380000000} & Tb''b\delta^{(6)}\delta''\delta \\
\frac{8370798286109273004877913}{1181019590663220000000} & Tb''b\delta^{(7)}\delta'\delta & \frac{-16862342598784223763053}{613516670474400000} & Tb''b'\delta^{(4)}\delta''\delta'
\end{array}
$$

$$
\begin{array}{llll}
\frac{7846771768524032514 5041}{6748683375218400000} & Tb''b'\delta^{(4)}\delta^{(3)}\delta & \frac{7062109150689282783 5233}{4217927109511500000} & Tb''b'\delta^{(5)}\delta''\delta \\
\frac{451321964277926191214 2229}{16871708438046000000 0} & Tb''b'\delta^{(6)}\delta'\delta & \frac{-182801942050921962966 9029}{101230250628276000000} & Tb^{(3)}b\delta^{(4)}\delta''\delta' \\
\frac{3215435022325180162621 1}{6748683375218400000} & Tb^{(3)}b\delta^{(4)}\delta^{(3)}\delta & \frac{114375282134342930831347 1}{50615125314138000000} & Tb^{(3)}b\delta^{(5)}\delta''\delta \\
\frac{968945774397383436869450 3}{506151253141380000000} & Tb^{(3)}b\delta^{(6)}\delta'\delta & \frac{-126923326442664511063993}{24102440625780000000} & Tb^{(3)}b'\delta^{(3)}\delta''\delta' \\
\frac{3064767243387062842645 9}{920275005711600000} & Tb^{(3)}b'\delta^{(4)}\delta''\delta & \frac{164518408082164312546529 9}{25307562657069000000} & Tb^{(3)}b'\delta^{(5)}\delta'\delta \\
\frac{13882785296081563127776 9}{7230732187734000000} & Tb^{(3)}b''\delta^{(3)}\delta''\delta & \frac{634702359450372316947259}{16871708438046000000} & Tb^{(3)}b''\delta^{(4)}\delta'\delta \\
\frac{-473109962874756576357 43}{1840550011423200000} & Tb^{(4)}b\delta^{(3)}\delta''\delta' & \frac{107868521037574748926787}{5061512531413800000} & Tb^{(4)}b\delta^{(4)}\delta''\delta \\
\frac{569591977630846094779621}{20246050125655200000} & Tb^{(4)}b\delta^{(5)}\delta'\delta & \frac{8237565721119697090349}{809842005026208000} & Tb^{(4)}b'\delta^{(3)}\delta''\delta \\
\frac{547949708460619385500073}{6748683375218400000} & Tb^{(4)}b'\delta^{(4)}\delta'\delta & \frac{7591704229758292563127}{149970741671520000} & Tb^{(4)}b''\delta^{(3)}\delta'\delta \\
\frac{316623867182060830615847}{20246050125655200000} & Tb^{(4)}b^{(3)}\delta''\delta'\delta & \frac{901231495195899807974983}{101230250628276000000} & Tb^{(5)}b\delta^{(3)}\delta''\delta \\
\frac{811764376867965846915707}{33743416876092000000} & Tb^{(5)}b\delta^{(4)}\delta'\delta & \frac{5010517066523392475 09}{9918699846000000} & Tb^{(5)}b'\delta^{(3)}\delta'\delta \\
\frac{794408731957024547027041}{33743416876092000000} & Tb^{(5)}b''\delta''\delta'\delta & \frac{1240318467076214946524429}{101230250628276000000} & Tb^{(6)}b\delta^{(3)}\delta'\delta \\
\frac{268246855030984122483697}{14461464375468000000} & Tb^{(6)}b'\delta''\delta'\delta & \frac{763343717601600647355311}{236203918132644000000} & Tb^{(7)}b\delta''\delta'\delta \\
\frac{-124466820073830843583}{1207537105500000} & Tb'b\delta^{(3)}\delta''\delta'\delta\alpha'' & \frac{-811892247963042565787}{8452759738500000} & Tb'b\delta^{(4)}\delta''\delta'\delta\alpha' \\
\frac{-5024769253737836707}{256144234500000} & Tb'b\delta^{(4)}\delta^{(3)}\delta'\delta\alpha & \frac{-31042377258207200509}{939195526500000} & Tb'b\delta^{(5)}\delta''\delta'\delta\alpha \\
\frac{-248609372040723083}{402512368500000} & Tb''b\delta^{(3)}\delta''\delta'\delta\alpha' & \frac{-86351280390679118581}{2817586579500000} & Tb''b\delta^{(4)}\delta''\delta'\delta\alpha \\
\frac{-1845656139030146677}{100628092125000} & Tb''b'\delta^{(3)}\delta''\delta'\delta\alpha & \frac{-8654634665276756611}{201256184250000} & Tb^{(3)}b\delta^{(3)}\delta''\delta'\delta\alpha \\
\frac{22420636199}{7387200} & T^2b'b\delta^{(5)}\delta''\delta & \frac{91565220149}{203148000} & T^2b'b\delta^{(6)}\delta'\delta \\
\frac{1108880152796373919}{270488311632000} & T^2b''b\delta^{(4)}\delta''\delta & \frac{815840987800876999}{270488311632000} & T^2b''b\delta^{(5)}\delta'\delta \\
\frac{-100512367423}{40629600} & T^2b''b'\delta^{(3)}\delta''\delta & \frac{157604654378837653}{30054256848000} & T^2b''b'\delta^{(4)}\delta'\delta \\
\frac{78084563531401661}{19320593688000} & T^2b^{(3)}b\delta^{(3)}\delta''\delta & \frac{983367807774000883}{270488311632000} & T^2b^{(3)}b\delta^{(4)}\delta'\delta \\
\frac{31473963893287721}{6440197896000} & T^2b^{(3)}b'\delta^{(3)}\delta'\delta & \frac{-4770998072911889}{1610049474000} & T^2b^{(3)}b''\delta''\delta'\delta \\
\frac{1017236448542048501}{270488311632000} & T^2b^{(4)}b\delta^{(3)}\delta'\delta & \frac{523706053025959297}{90162770544000} & T^2b^{(4)}b'\delta''\delta'\delta \\
\frac{386807744976622463}{270488311632000} & T^2b^{(5)}b\delta''\delta'\delta & \frac{742729403}{677160} & T^2b'b\delta^{(3)}\delta''\delta'\delta\alpha \\
\frac{6662616373566101}{19320593688000} & T''Tb'b\delta^{(3)}\delta''\delta & \frac{-93154238427865849}{22540692636000} & T''Tb'b\delta^{(4)}\delta'\delta \\
\frac{-1552126807894351}{1073366316000} & T''Tb''b\delta^{(3)}\delta'\delta & \frac{52065275724588347}{6440197896000} & T''Tb''b'\delta''\delta'\delta \\
\frac{30060539578788199}{19320593688000} & T''Tb^{(3)}b\delta''\delta'\delta & \frac{144086319689721929}{67622077908000} & T^{(4)}Tb'b\delta''\delta'\delta \\
\frac{1190621242967}{1940598000} & T^3b'b\delta^{(3)}\delta''\delta & \frac{-83824942877}{485149500} & T^3b'b\delta^{(4)}\delta'\delta \\
\frac{471463088837}{4608920250} & T^3b''b\delta^{(3)}\delta'\delta & \frac{4544925946691}{12290454000} & T^3b''b'\delta''\delta'\delta \\
\frac{-8617452102727}{36871362000} & T^3b^{(3)}b\delta''\delta'\delta & \frac{-88001317417}{62073000} & T''T^2b'b\delta''\delta'\delta \\
\frac{-49278745789}{646866000} & T^4b'b\delta''\delta'\delta. & &
\end{array}
$$

There are 51 ghost number 4 terms. Only one term depending on W is set to zero. The rest 50 terms are as follows:

$$
\begin{array}{llll}
\frac{8940122032541126891604 79}{268966882800000000} & b''b'b\delta^{(5)}\delta^{(4)}\delta''\delta' & \frac{40194451106867620759 5847}{427901859000000000} & b''b'b\delta^{(5)}\delta^{(4)}\delta^{(3)}\delta \\
\frac{40752983873787784781775541}{1412076134700000000 0} & b''b'b\delta^{(6)}\delta^{(3)}\delta''\delta' & \frac{28258929465723947578300037}{7060380673500000000 0} & b''b'b\delta^{(6)}\delta^{(4)}\delta''\delta \\
\frac{39761331777147288534 39929}{313794696600000000 0} & b''b'b\delta^{(6)}\delta^{(5)}\delta'\delta & \frac{136545858316535273628988667}{49422664714500000000} & b''b'b\delta^{(7)}\delta^{(3)}\delta''\delta \\
\frac{10172277510601489892564111}{41185553928750000000} & b''b'b\delta^{(7)}\delta^{(4)}\delta'\delta & \frac{43376645108126177448410147}{26358754514400000000} & b''b'b\delta^{(8)}\delta^{(3)}\delta'\delta \\
\frac{88568028449146814148822683}{197690658858000000000} & b''b'b\delta^{(9)}\delta''\delta'\delta & \frac{-24954894000136167461843 93}{12551787864000000000} & b^{(4)}b'b\delta^{(4)}\delta^{(3)}\delta''\delta' \\
\frac{-40920180369199307232545177}{18827681796000000000} & b^{(4)}b'b\delta^{(5)}\delta^{(3)}\delta''\delta & \frac{-326733424846430799735 59}{24515210671875000} & b^{(4)}b'b\delta^{(5)}\delta^{(4)}\delta'\delta \\
\frac{-61682288510900776644 87283}{26896688280000000000} & b^{(4)}b'b\delta^{(6)}\delta^{(3)}\delta'\delta & \frac{-502424830932040850726 93629}{43931257524000000000} & b^{(4)}b'b\delta^{(7)}\delta''\delta'\delta \\
\frac{-24184082584153779002 40491}{20919646440000000000} & b^{(5)}b'b\delta^{(4)}\delta^{(3)}\delta''\delta & \frac{-14721008866519644234 31343}{8716519350000000000} & b^{(5)}b'b\delta^{(5)}\delta^{(3)}\delta'\delta \\
\frac{-126093110562930053290 10411}{104598232200000000000} & b^{(5)}b'b\delta^{(6)}\delta''\delta'\delta & \frac{-167948593256420880617 03839}{188276817960000000000} & b^{(6)}b'b\delta^{(4)}\delta^{(3)}\delta'\delta \\
\frac{-103421685992912472631 98641}{104598232200000000000} & b^{(6)}b'b\delta^{(5)}\delta''\delta'\delta & \frac{26532493353233040719}{92217216960000} & b^{(6)}b^{(3)}b\delta^{(3)}\delta''\delta'\delta \\
\frac{-733102719066632691 53}{95632669440000} & b^{(7)}b'b\delta^{(4)}\delta''\delta'\delta & \frac{-11202877685146493099}{40985429760000} & b^{(8)}b'b\delta^{(3)}\delta''\delta'\delta \\
\frac{969576920701574621683}{7263766125000000} & b''b'b\delta^{(5)}\delta^{(3)}\delta''\delta'\delta\alpha & \frac{969576920701574621683}{4358259675000000} & b^{(3)}b'b\delta^{(4)}\delta^{(3)}\delta''\delta'\delta\alpha \\
\frac{-6922466437361968939693}{815051160000000 0} & Tb''b'b\delta^{(4)}\delta^{(3)}\delta''\delta' & \frac{1231279142989406788161469}{3137946966000000000} & Tb''b'b\delta^{(5)}\delta^{(3)}\delta''\delta \\
\frac{-4587586135891723505 3603}{784486741500000000} & Tb''b'b\delta^{(5)}\delta^{(4)}\delta'\delta & \frac{1354999687164657192816391}{3137946966000000000} & Tb''b'b\delta^{(6)}\delta^{(3)}\delta'\delta \\
\frac{6799671218996227038570623}{21965628762000000000} & Tb''b'b\delta^{(7)}\delta''\delta'\delta & \frac{366684281907420362457 91}{2091964644000000000} & Tb^{(3)}b'b\delta^{(4)}\delta^{(3)}\delta''\delta \\
\frac{4550158381959521999637983}{4706920449000000000} & Tb^{(3)}b'b\delta^{(5)}\delta^{(3)}\delta'\delta & \frac{452635413724118337879361}{448278138000000000} & Tb^{(3)}b'b\delta^{(6)}\delta''\delta'\delta \\
\frac{285551042647987065597089}{627589393200000000} & Tb^{(3)}b''b\delta^{(4)}\delta^{(3)}\delta'\delta & \frac{186740014116478534834 07}{135841860000000000} & Tb^{(3)}b''b\delta^{(5)}\delta''\delta'\delta \\
\frac{712530383295049840843673}{627589393200000000} & Tb^{(3)}b''b'\delta^{(4)}\delta''\delta'\delta & \frac{110108432707487851779 61}{116220258000000000} & Tb^{(4)}b'b\delta^{(4)}\delta^{(3)}\delta'\delta \\
\frac{1062809816042147714999159}{784486741500000000} & Tb^{(4)}b'b\delta^{(5)}\delta''\delta'\delta & \frac{1047526394937532878927871}{6275893932000000000} & Tb^{(4)}b''b\delta^{(4)}\delta''\delta'\delta \\
\frac{892672453706043323093 3}{78448674150000000} & Tb^{(4)}b''b'\delta^{(3)}\delta''\delta'\delta & \frac{5667634504872537413983}{156897348300000000} & Tb^{(4)}b^{(3)}b\delta^{(3)}\delta''\delta'\delta \\
\frac{8857370932893413197263 1}{784486741500000000} & Tb^{(5)}b'b\delta^{(4)}\delta''\delta'\delta & \frac{172495372529594372064287}{1961216853750000000} & Tb^{(5)}b''b\delta^{(3)}\delta''\delta'\delta \\
\frac{6463687064298599698314 7}{1568973483000000000} & Tb^{(6)}b'b\delta^{(3)}\delta''\delta'\delta & \frac{2373366207352179189616 7}{3137946966000000000} & T^2b''b'b\delta^{(4)}\delta^{(3)}\delta'\delta \\
\frac{-4812734794405584883421}{285267906000000000} & T^2b''b'b\delta^{(5)}\delta''\delta'\delta & \frac{-215727295354745513 3}{25357147200000} & T^2b^{(3)}b'b\delta^{(4)}\delta''\delta'\delta \\
\frac{1102894925261501388235 7}{784486741500000000} & T^2b^{(3)}b''b\delta^{(3)}\delta''\delta'\delta & \frac{-11617696545038738995243}{1568973483000000000} & T^2b^{(4)}b'b\delta^{(3)}\delta''\delta'\delta \\
\frac{1397336113203162961517}{34866077400000000} & T''Tb''b'b\delta^{(3)}\delta''\delta'\delta & \frac{-5041067164155497640971}{784486741500000000} & T^3b''b'b\delta^{(3)}\delta''\delta'\delta.
\end{array}
$$

Finally the 4 ghost number 5 terms are:

$$
\begin{aligned}
&\frac{-3466641462380311323335551109341621}{1286130334232246580000000000} && b^{(3)}b''b'b\delta^{(5)}\delta^{(4)}\delta''\delta'\delta \\
&\frac{2862389756866155092068795501394 3}{13919159461387950000000000} && b^{(3)}b''b'b\delta^{(6)}\delta^{(3)}\delta''\delta'\delta \\
&\frac{3955472519267611138671522268507 9}{114833065556450587500000000} && b^{(5)}b''b'b\delta^{(4)}\delta^{(3)}\delta''\delta'\delta \\
&\frac{53938039562785304406617940923 9}{801926882549100000000000} && Tb^{(3)}b''b'b\delta^{(4)}\delta^{(3)}\delta''\delta'\delta.
\end{aligned}
$$

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